SHEAR DEFORMATION IMPACT ON DEFLECTION OF COMPOSITE BRIDGE GIRDERS

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Abstract
The aim of the article is to present a study focused on the deformation of composite steel and concrete bridge girders, especially on effects of shear forces. Additional deflections due to vertical shear are usually neglected in design procedure of composite girders. However, if shear forces may produce more significant influence on the resulting vertical deformation of these structures, supplementary values have not been still sufficiently examined. In addition, no practical rules are available for estimation of this effect in preliminary design of steel-concrete composite bridges. Parametrical study dealing with this assumption is also given in the contribution. Finally, some practical conclusions concerning the estimation of shear deformations are suggested.

1 INTRODUCTION

Shear deformations in typical steel welded built up girders are obviously assumed negligible in comparison to secondary impact on the overall deflection of the bridge. The article would present the study focusing on the influence of vertical shear deformation in the case of steel girders combined with concrete slab into a composite structure. A reduced steel part is required in comparison to classical steel structure. Thus, the ratio of girder height to its length can be greater. The dominant shear flow is supposed to be carried by steel web. Therefore, the higher influence of shear deformation can be expected. Eurocode 4 [1] provides no estimation concerning of a vertical shear influence or any recommendations for predetermination of this influence on flexural behaviour. The deflection should generally be determined from the more complex relation of the beam theory

\[ \delta_x(x) = \frac{M_x(x)}{E_a \cdot I_y(x)} + \frac{V_z(x)}{G_a \cdot A_v(x)} \]  

(1)

where: \( \delta_x(x) \) is actual vertical deflection, \( M_x(x) \) and \( V_z(x) \) represent bending moment and vertical shear force, value \( I_y(x) \) is second moment of area of effective composite section, \( A_v(x) \) is shear area of cross-section, and finally \( E_a \) and \( G_a \) are modulus of elasticity and shear modulus of steel, respectively.

When I-shaped girders in steel bridges are used, deformation due to vertical shear is usually less then 10% of flexural value. In practical cases, only cross-section of a steel web \( A_w \) is taken as the shear area \( A_v \) in the above formulae.

To confirm all these assumptions for steel-concrete composite girders, further experimental or numerical models and data are necessary. A combined cross-section with wide concrete flange connected to the steel girder can produce particular differences. The following parametrical study provides some rather remarkable results. Firstly, determination of the vertical shear influence on overall deflection of the beam is studied. Secondly, practical conclusions for a simple estimation of this effect are recommended. Our study presented in the following paragraph considers the simply supported composite steel and concrete beams subjected to sagging bending moment. The experimental measurement on specimens of composite beams under transverse load producing tension in concrete was presented in the paper[2]. Relatively higher values of shear deformation in the hogging moment region were concluded.
2 PARAMETRIC STUDY DETAILS

2.1 Geometrical characteristic of bridge girders
The real cross-sections of a road bridge have been considered in the parametric study to get closer to the real geometrical characteristics of composite girders, Figure 1.

![Figure 1 Bridge cross-section](image)

Seven different lengths of bridges were considered in the study. The bridge girders at the Figure 2 had spans L of 20, 25, 30, 35, 40, 45 and 50 metres. The height of steel girder web $h_w$ was derived as a ratio of the corresponding span. Six ratios $h_w/L$ were taken into account, specifically 1/16, 1/18, 1/20, 1/22, 1/24 and 1/26. Consequently, 42 composite girders were then recalculated in the study. A concrete deck thickness $t_c = 250$ mm and effective slab width $b_c = 3.0$ m were unvarying for all the girders.

Loads according to the design code[3] were taking into account for determination of the other dimensions of steel welded girders. Concrete casting without any additional temporary support of steel beams was considered. Thus, the steel girders carry a self-weight of the structure including concrete deck. After concrete hardening, the following long-term actions are transferred through the composite structure. The most important live load was found as standard scheme I given in the code[3].

The sections of steel beams have been determined to ensure the same level of load carrying capacity for all composite girders. Design criteria have required obtaining exactly 70% of ultimate plastic beam moment. In addition, the value of 75% of steel yield stress was guaranteed in top steel flange in building
stage during concrete casting. On the basis of the above two requirements, 42 cross-sections with bottom flanges from 24 x 240 mm to 60 x 750 mm were determined. Limit dimensions 12 x 200 mm of the top flange was given by construction requirements. The largest top flange had not to be greater as 40 x 400 mm.

The approximate thickness of steel web was calculated from the relationship

$$t_w = 7 + 3 \cdot h_w$$  \hspace{1cm} (2)

where: $t_w$ is thickness of steel web in [mm] and $h_w$ its height also in [mm].

During the design, we have met in some cases problems with classification of composite cross-section and obtained inadequate web shear resistance. Therefore, steel webs in this design situation were taken 1 mm thicker for 21 girders with height $h \leq L / 22$. Girders with height $h \geq L / 20$ had thickness 2 mm greater in comparison with the basic value given by the above formulae. The steel quality for built-up welded unsymmetrical girders was S 355. Concrete of class C 30/37 was considered for bridge deck.

Serviceability criteria for stresses and elastic behaviour of girders under service load have been also checked for all girders. Shear leg in concrete deck, creep and shrinkage have been taken into account in this limit state. The limit values of deflections have been controlled, as well. As a result, only the girders of real layout were a subject of our parametrical study.

2.2 Parametric study on the girders

ANSYS program was used for numerical modelling of girders. A opportunity of parametric inputs implemented into this finite element system was advantage. For simulation of deck volume, the concrete part of cross-section in the numerical model was divided into SOLID65 finite elements. Both flanges of steel girder consisted of volume SOLID45 elements. For a mesh idealising steel web, shell finite elements SHELL63 were used. More details about properties of the elements and the program procedures can be found in reference[4].

Connection between top steel flange and concrete slab was modelled as rigid. Thus, influence of slip on vertical deflection was eliminated. Static system in accordance to the Figure 2 was considered. The symmetry of the model was used for saving memory space and computing time, Figure 3.

Figure 3 An example of numerical model  
(a half of the girder with span of $L = 20$ m and web height of $h_w = L / 20 = 1.5$ m)

Deflection of all 42 girders in the middle of the span was calculated for two load cases. Firstly, the girder was subjected to the uniformly distributed load on the top concrete surface, Figure 4a. A force in the middle of the span with contact area according to Figure 4b represented second load case in the parametrical study.
3 RESULTS ASSESSMENT

For assessment of influence of vertical shear on deflection, the following percentage ratio was chosen

\[
\frac{\delta_{z,\text{tot}} - \delta_{z,M}}{\delta_{z,M}} \cdot 100\% \qquad (3)
\]

where: \(\delta_{z,\text{tot}}\) is overall deflection including shear deformation resulting from ANSYS, \(\delta_{z,M}\) deformations of the girder due to bending moments without shear influence.

The impact of shear deformation on deflections expressed in percentage for all 42 girders is given at the Figure 5. Results for both the uniformly distributed load and single force in the mid-span are illustrated, too.

At the Figure 6, the impact of shear deformation is illustrated for uniformly distributed load applied on the top surface of concrete slab. The curves at the Figure 7 express the same relationship in the case of a concentrated force in the middle of the span. It is evident from the graphs that in the case of obvious heights of bridge girders, the influence of shear is from 5 to 9 percent of deflection due to bending moment alone. The differences are very similar for both load cases considered.
From the three graphs presented above, it is also apparent the negligible shear effects height of the steel web height. In fact, the difference between total deflection and only moment one is above 1.2% for all calculated girders, practically insensitive on the web heights.

In the Table 1 are given numerical values, which correspond to the difference in percentage of deflection calculated according to the function (1) in comparison with ANSYS data ($\delta_{z, \text{tot}, \text{ANSYS}}$). The cross-section of steel web $A_v = A_w$ was considered as shear area in by-hand calculation of deflections $\delta_{z, M+V, (1)}$.

Data in the table demonstrate small differences, if uniformly distributed load is taken into account. It means that beam theory given in formulae (1) using steel web as shear area is accurate enough for this type of loading.

However, from lower part of the Table 1 is evident that for concentrated force, deflections given by the beam theory are overestimated. The differences are in the range form 5% to 16% and are greater for girders with shorter spans and lower heights of steel beams. For example, if the girder of span $L = 30$ metres is considered and its web height will be for instance $h_w = L / 24 = 1.25$ m, deflection given by relation (1) will be nearly 11% greater than the deformations given by ANSYS. On the other hand, if deflection will be calculated without consideration of shear effect, deformation would be underestimated. However, the difference in that case is expected to be about 7.5% (Figure 7).
Table 1  Difference in percentage of deflection calculated according to (1) when $A_v = A_w$

<table>
<thead>
<tr>
<th>$A_v = A_w$</th>
<th>$h_w$</th>
<th>Load</th>
<th>Span L [m]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>$h_w = L / 16$</td>
<td>$g$</td>
<td></td>
<td>0.13%</td>
</tr>
<tr>
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</tr>
<tr>
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<td>$g$</td>
<td></td>
<td>1.53%</td>
</tr>
<tr>
<td>$h_w = L / 24$</td>
<td>$g$</td>
<td></td>
<td>1.91%</td>
</tr>
<tr>
<td>$h_w = L / 26$</td>
<td>$g$</td>
<td></td>
<td>2.53%</td>
</tr>
<tr>
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<td>$F$</td>
<td></td>
<td>11.10%</td>
</tr>
<tr>
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</tr>
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<td>$F$</td>
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<td>$h_w = L / 26$</td>
<td>$F$</td>
<td></td>
<td>16.11%</td>
</tr>
</tbody>
</table>

4 CONCLUSIONS AND RECOMMENDATIONS

4.1 Concluding comments

Based on the evaluation of presented results it is apparent relatively small influence of shear on vertical deflection as generally expected. Influence of shear deformations in composite girders has been observed very similar as in the case of steel bridge beams. Nevertheless, more appropriate design procedure of bridge structures should take into account shear impact on deflection, as well. Moreover, other effects, like concrete rheology, are to be considered in design of girders. Therefore, it is suitable to examine every effect with the best accuracy for obtaining appropriate resulting values.

The shape of curves presented at both Figure 6 and Figure 7 is almost linear. Furthermore, a variation of the results due to height of steel cross-section was found negligible. The linear relationship between the deformation due to shear and the span of the girder should be easily acceptable. Resulting deflection can be calculated by multiplication of deflection due to moment $\delta_{z, M} = k_\delta \cdot \delta_{z, M}$ (4)

where: $k_\delta$ is a “shear deflection factor”, $\delta_{z, M}$ is deformation of the girder due to bending moments without shear influence as a resulting solution of differential equation $\delta_{z, M}(x) = \frac{M_p(x)}{E_g \cdot l_1(x)}$ (5)

where all symbols are already known from equations (1) and (4).

According to results of the parametric study, linear regression analysis could be applied. The following relations have been found for shear deflection factor as a linear function of the span $k_{\delta, q} = 1.0994 + 0.001052 \cdot L$ $k_{\delta, F} = 1.1075 + 0.001056 \cdot L$ (6)

where: $k_{\delta, q}$ and $k_{\delta, F}$ represent shear deflection factor for uniformly distributed load and for single force in the middle of the span, respectively; $L$ is the span of composite girder in [m].

Differences between two formulas given in (6) are rather small. From the practical point of view, it seems to be more convenient if only one factor $k_\delta$ could exist. In addition, live load actions in the service are a
random values and highly variable combinations of distributed load and concentrated forces. Therefore, for calculation of the shear deflection only one factor may be used. We would suggest the following more general relation

\[ k_\delta = 1.1 + 0.001 \cdot L \]

(7)

where: \( k_\delta \) is shear deflection factor and \( L \) the span of composite girder in [m].

![Figure 8: Shear deflection factor](image)

From the Figure 8, it is evident, that errors due to introducing only this single factor are insignificant. For 42 considered girders and two load cases, deflections calculated according to formulas (4), (5) and (7) match very well to the ANSYS results. The differences in alternative values are less than 1%.

Table 1 shows that if steel web is considered as the only shear area for calculation of a deflection increment due to shear, overestimated deformations can be sometimes received. The contribution of the concrete slab in shear transferring will be a subject of our next research.

4.2 Remarks

It should be also emphasized, that several common commercial programs have implemented incorrect algorithms for calculation of shear area \( A_v \) for composite steel and concrete cross-sections. When consideration of shear deformation is imposed in such calculation of a member by finite element model in these programs, different results can be obtained. The differences of calculated shear areas in the FEM programs are sometimes remarkable. The parametric study in the paper confirmed influence of vertical shear less than 10%. Hence, differences in deflections produced by FEM programs by different numerical model of a composite cross-section are actually smaller than for shear effects.

Nevertheless, to avoid this problem a suggested method for shear deformation assessment can be used. If such FEM system is used, the more exact values of deflection may be obtained when the influence of shear deformation is switched off during calculation. Deflections given by such analysis have to be than modified by the shear deflection factor \( k_\delta \) specified in (7). This factor depends only on the span of the girder. Deformation can be then calculated with sufficiently precision.

4.3 Acknowledgement

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5 REFERENCES


